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A Mathematical Approach to the Stability of Electroencephalography Signals Model Using Integral Equations During Epileptic Seizures

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Abstract

In this study, the mathematical stability of EEG signal models represented by the integral equation during an epileptic seizure is investigated. The modelling of the EEG signals should be done with utmost accuracy to understand the actual nature of the brain dynamics and to enhance seizure prediction approaches. We investigate the stability of such models by applying Lyapunov's stability theorem and show that stability is powerfully dependent on parameters such as seizure duration and intensity, along with neural connectivity. We demonstrate that some of the parameter regimes yield stable behaviour, while others give rise to instability, making EEG interpretation complicated during epileptic seizures. By comparing integral equation-based models with current approaches, their advantages are highlighted in capturing the transient dynamics of epileptic activity. This work focuses on developing solid mathematical frameworks that will allow for real-time EEG monitoring. It contributes to the wider knowledge on the dynamics of seizures in the brain and will have practical implications for the development of neurological diagnostic and therapeutic tools.

Key words: Asymptotically stable, EEG signals, Integral equation, Lyapunov's stability.

1. Introduction

Epilepsy is a neurological disorder that involves several disorders characterized by abnormal electrical discharges in the brain [1]. The disorder results from excessive and unpredictable discharge of specific groups of neurons; the process has been termed a miniature brainstorm. Such electrical storms may cause a range of symptoms, from sudden, brief muscle contractions to generalized convulsions [2]. Some people also become unconscious during such seizures; it may last from a short duration to a long one, and the intensity may vary. Epilepsy may be confined to one area of the brain or may be more generalized to involve the whole brain. Depending on which part of the brain has been involved, symptoms may range from changes in movement, sensation, or emotions, and state of consciousness, or even in behaviours, and are collectively referred to as epileptic seizures. Such complexities are important to understand for the elaboration of effective treatment and management strategies in patients suffering from this condition [3].

Electroencephalography, or EEG, is a method of recording electrical signals generated by the brain. it is essential in epilepsy diagnosis, documenting evidence of seizure disorders and helping classify various types of seizures [4]. Generally, EEG is used in cases observed with abnormal brain activities occurring during epileptically events. This process involves attaching electrodes to the scalp, which allows for a non-invasive retrieval of signals from the brain. The particular patterns of activity recorded and the source within the brain will give the physician important information about which medication works best for certain types of epilepsy [5]. If seizures are not controllable by medication, then patients can resort to surgery, which removes the pieces of brain tissue where the seizures originate. Because of this, EEG becomes critical in demarcating the precise locations of such damaged tissues for appropriate interventions, therefore dramatically enhancing outcomes for the patients [6].

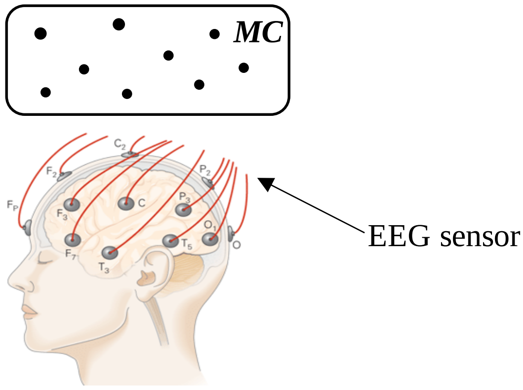

Figure1. EEG Projection

Fuzzy Topographic Topological Mapping (FTTM) represents an innovative approach in efforts to solve the neuromagnetic inverse problem. This model consists of four elements, which are homoeomorphic from each other: magnetic contour plane (*MC*),

base magnetic plane (*BM*), fuzzy magnetic field (*FM*), and topographic magnetic field (*TM*). It has been generalized for this model in the literature. There has also been an application of a similar concept of topological mapping for navigation and localization to support visually impaired individuals. As it follows in the paper by [7], a new technique was proposed for mapping high-dimensional EEG signals into *MC* low-dimensional space. In general, the whole scheme of the new model incorporates steps as follows. First, there is a need for the flattening of the EEG-a process for the conversion of three-dimensional data into two-dimensional format by strategically placing sensors on a patient's head relative to the EEG signals. This method of flattening will be able to preserve both the magnitude and orientation of the surface effectively, as shown in Fig.1. The second step is the processing of EEG data using Fuzzy C-Means (*FCM*).

The application of mathematical modelling was vital in the analysis of these EEG signals, as this provides a systematic interpretation of the complex electrical activities taking place in the human brain during the event of an epileptic seizure. Barja 2024, resorts to a different way of understanding and modelling these EEG signals by formulating integral equations. It captures the complex dynamics of brain activity at critical seizure episodes and provides insight into the mechanisms of seizure propagation. Further, Barja reformulated traditional models to outline the potential that integral equations may hold for improving seizure detection and classification [9]. This innovative perspective opens a new dimension, where not only is there a mathematical framework established for the interpretation of EEG data, but new perspectives toward diagnosis and therapeutic strategy are opened. These results confirm that sophisticated mathematical modelling plays an important role in neuroscience, with clinical implications. This will help in the better understanding of epilepsy and how it affects the functions of the brain.

2. Related Work and Preliminary Considerations

In this section, we will explore the existing literature relevant to our study, highlighting key studies and findings that inform our work while also presenting fundamental definitions and essential theorems that underpin the concepts discussed. By examining prior study and establishing a clear theoretical framework, we aim to provide a comprehensive context for our study and clarify the significance of our contributions to the field. Furthermore, as mentioned in the introduction, Barja 2024 developed a new model to describe EEG signals during an epileptic seizure as an integral equation of the following form:

$$\phi(t) = \int_{t}^{\infty} K(\tau)\, v(t-\tau)\mu(\tau)d\tau \qquad (2.1)$$

The equation (2.1), with its kernel function $K(\tau)$, pre-seizure signal $v(t-\tau)$, and seizure function $\mu(\tau)$, served as a key tool for unlocking the connection between the observed EEG signal $\phi(t)$ during a seizure and the underlying brain activity $\mu(\tau)$. By solving (2.1), we could estimate the seizure activity using the measured EEG signal. This breakthrough had profound implications, allowing us to pinpoint and localize the source of seizures within the brain, as demonstrated in Fig 2.

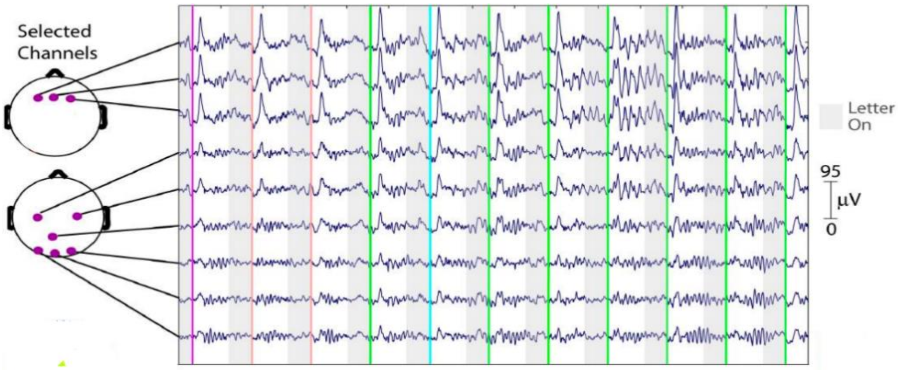

Figure2. Brain activity captured by EEG during an epileptic episode

Additionally, the prior integral equation (2.1) was implemented in MATLAB, yielding unambiguous results in the referenced study as shown in Fig 3.

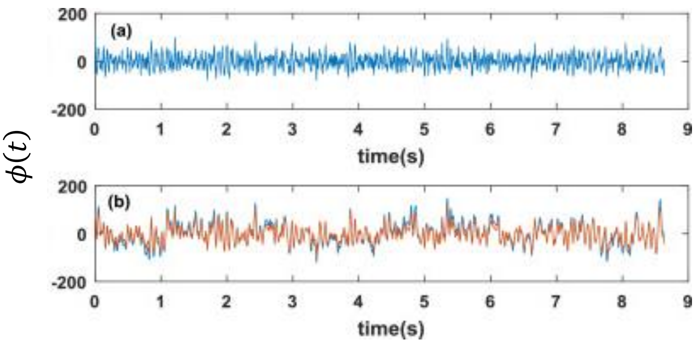

Figure3. Integral Equation Mapping of EEG Signals During a Seizure

2.1 Definition [10]: Let $\Upsilon(t)$ represent the state of the system at time t. The evolution of the system is described by a set of equations, which can be written in two forms:

1. Continuous-Time Dynamic System: $\frac{d}{dt}\Upsilon(t) = \tau(\Upsilon(t), t)$, where $\tau: \mathbb{R}^n \times \mathbb{R} \longrightarrow \mathbb{R}^n$ is a continuous function defining the dynamics of the system.
2. Discrete-Time Dynamic System: $\Upsilon(k+1) = \varphi(\Upsilon(k), k)$, where $\varphi: \mathbb{R}^n \times \mathbb{Z} \longrightarrow \mathbb{R}^n$ is a function that defines the state transition at discrete time steps k.

2.2 Definition [11]: A solution $\Upsilon(t)$ of a nonlinear dynamical system described by an integral equation is said to be Lyapunov stable if, for any $\varepsilon > 0$ there exists a $\delta > 0$ such that, if the initial condition $\Upsilon(0)$ is within δ of an equilibrium point Υ_e, the solution $\Upsilon(t)$ remains within ε of Υ_e for all $t \geq 0$. Mathematically, this can be expressed as:

$$\forall t \geq 0\ ;\ \|\Upsilon(0) - \Upsilon_e\| < \delta \Longrightarrow \|\Upsilon(t) - \Upsilon_e\| < \varepsilon$$

2.3 Definition [12]: Suppose the following integral equation:

$$\Upsilon(t) = \Upsilon_0 + \int_0^t \tau(\Upsilon(s), s)ds,$$

where τ is a continuous function, if there exists a continuously differentiable function $L: \mathbb{R}^n \longrightarrow \mathbb{R}$ ((known as the Lyapunov function) such that:

1. Positive Definite: $L(\Upsilon) > 0$ for all $\Upsilon \neq \Upsilon_e$ and $L(\Upsilon_e) = 0$.
2. Negative Definite Derivative: The time derivative $\dot{L} = \nabla L \cdot \tau(\Upsilon(t), t) < 0$ for all Υ in a neighbourhood of Υ_e.

Then, the equilibrium point Υ_e is globally asymptotically stable. This means that not only is the system stable, but all trajectories will eventually converge to Υ_e as time approaches infinity.

2.4 Definition [13]: A steady-state solution is a solution to a system (in the case of an integral equation) that does not change over time, In mathematical terms, a steady-state solution $\varphi_0(t)$ satisfies the integral equation $\int_\alpha^\beta K(t,x)\nu(x)dx = \varphi(t)$ in such a way that it remains constant or converges to a fixed value as $\alpha \longrightarrow \beta$. The solution $\varphi_0(t)$ satisfies $\varphi_0(t) = \varphi(t)$ for all t.

2.5 Definition [14]: Let $K(t,s)$ be a kernel function on region R. We say that K satisfies a Lipschitz condition with respect to the second variable (s) if there exists a positive constant L such that: $|K(t,s_1) - K(t,s_2)| \leq L|s_1 - s_2|$ for all (t,s_1) and (t,s_2) in R.

2.6 Banach's Fixed-Point Theorem [15]: Let (X, d) be a complete metric space, and let $T: X \longrightarrow X$ be a contraction mapping, meaning there exists a constant $0 \leq k < 1$ such that for all $x, y \in X$: $d\big(T(x), T(y)\big) \leq k \cdot d(x, y)$. Then, the following statements hold:

1. Existence of a Fixed Point: There exists a unique point $x^* \in X$ such that $T(x^*) = x^*$.
2. Convergence to the Fixed Point: For any point $x_0 \in X$, the sequence defined by $x_{n+1} = T(x_n)$ converges to the fixed point x^* as $n \longrightarrow \infty$.

3. Methods and Results

The objective of this study is to analyse the stability of integral equation 2.1 models that describe EEG signals during epileptic seizures. The following methodology was adopted to achieve this:

1. Model Definition: We define a generalized integral equation to model the EEG signal during epileptic seizures. This equation takes the form of an integral equation system where the EEG signal $\phi(t)$ depends on a kernel function $K\big(\tau, \phi(t)\big)$, the pre-seizure signal $v(t)$, and the seizure dynamics $\mu(\tau)$.
2. Assumptions on System Properties: The kernel function $K\big(\tau, \phi(t)\big)$, as well as the functions $v(t)$ and $\mu(\tau)$, are assumed to be continuous and, in some cases, satisfy properties such as boundedness and differentiability. These assumptions allow us to explore the stability of the solution $\phi(t)$.
3. Linearization and Perturbation Analysis: To analyse stability, we linearize the nonlinear terms of the integral equation 2.1 around a steady-state solution. We then introduce small perturbations into the system and observe how the solution responds, focusing on Lyapunov and asymptotic stability.

4. Proof of Stability: We derive theorems that provide sufficient conditions for the existence, uniqueness, boundedness, and stability of the solution to the integral equation 2. 1..

Following the outlined methodology, we present the main results of our study in the form of several key theorems, each accompanied by a rigorous mathematical proof to establish the stability properties of the integral equation 2.1 model.

3.1 Theorem: Let $K(\tau, \phi(t))$ be continuous with respect to both τ and $\phi(t)$, and let $\upsilon(t)$ and $\mu(\tau)$ be continuous and bounded functions. Then there exists a unique continuous solution $\phi(t)$ to the integral equation:

$$\phi(t) = \int_t^\infty K(\tau, \phi(t))\, \upsilon(t-\tau)\mu(\tau)d\tau$$

Proof.

Assume that $K(\tau, \phi(t))$ is continuous with respect to both τ and EEG signals $\phi(t)$, and satisfies a Lipschitz condition with respect to $\phi(t)$.

we will define the integral operator T for any $\phi(t)$ during epileptic seizure as:

$$T[\phi](t) = \int_t^\infty K(\tau, \phi(t))\, \upsilon(t-\tau)\mu(\tau)d\tau$$

Let $\phi_1(t)$ and $\phi_2(t)$ be two continuous functions. We need to show that T is a contraction mapping, i.e., for some constant $L < 1$: $\|T[\phi_1] - T[\phi_2]\| \le L\|\phi_1 - \phi_2\|$

Now $T[\phi_1](t) - T[\phi_2](t) = \int_t^\infty \left[K(\tau, \phi_1(t)) - K(\tau, \phi_2(t))\right] \upsilon(t-\tau)\mu(\tau)d\tau$

Since $K(\tau, \phi(t))$ is Lipschitz continuous with respect to $\phi(t)$, there exists a constant $L_K > 0$ such that: $\left|K(\tau, \phi_1(t)) - K(\tau, \phi_2(t))\right| \le L_K|\phi_1(t) - \phi_2(t)|$. Thus, we have $|T[\phi_1](t) - T[\phi_2](t)|$

$$\le L_K|\phi_1(t) - \phi_2(t)| \int_t^\infty |\upsilon(t-\tau)||\mu(\tau)|d\tau\,.$$ hence T is a contraction.

By assumption, there exist constants M_υ and M_μ such that $|\upsilon(t)| \le M_\upsilon$ and $|\mu(\tau)| \le M_\mu$ for all t and τ. Therefore, the integral can be bounded as follows:

$$\int_t^\infty |\upsilon(t-\tau)||\mu(\tau)|d\tau \le M_\upsilon M_\mu \int_t^\infty d\tau$$

Since the integral $\int_t^\infty d\tau$ converges for appropriate choices of $\upsilon(t)$ and $\mu(\tau)$, we obtain:

$$|T[\phi_1](t) - T[\phi_2](t)| \le L_K M_\upsilon M_\mu |\phi_1(t) - \phi_2(t)|$$

Thus, the operator T satisfies the contraction condition with $L = L_K M_\upsilon M_\mu$. If $L < 1$, and since T is a contraction mapping, 2.4 theorem guarantees the existence of a unique fixed point $\phi(t)$ that satisfies: $\phi(t) = T[\phi](t)$. This fixed point is unique solution to the original integral equation 2.1 for EEG signals during the seizure, as required.

3.2 Theorem: let $\upsilon(t)$ and $\mu(\tau)$ be continuous and bounded functions and let the kernel function $K\big(\tau,\phi(t)\big)$ satisfy a Lipschitz condition with respect to EEG signals $\phi(t)$. If L_K is sufficiently small, then the solution $\phi(t)$ of the integral equation 2.1 is Lyapunov stable.

Proof.

Consider two solutions $\phi_1(t)$ and $\phi_2(t)$ corresponding to slightly different initial conditions. Since $K\big(\tau,\phi(t)\big)$ satisfy a Lipschitz condition with respect to EEG signals $\phi(t)$, then there exists a constant $L_K > 0$ such that for all t; $\big|K\big(\tau,\phi_1(t)\big) - K\big(\tau,\phi_2(t)\big)\big| \le L_K |\phi_1(t) - \phi_2(t)|$

Now, $|\phi_1(t) - \phi_2(t)| = \left|\int_t^\infty \big[K\big(\tau,\phi_1(t)\big) - K\big(\tau,\phi_2(t)\big)\big]\, \upsilon(t-\tau)\mu(\tau)d\tau\right|$

$$\leq L_K \int_t^{\infty} |\phi_1(t) - \phi_2(t)|\, |\upsilon(t - \tau)||\mu(\tau)| d\tau \quad \text{(by Using the Lipschitz condition on } K(\tau, \phi(t))\text{)}$$

Since L_K is small and $\upsilon(t)$, $\mu(\tau)$ are bounded, this implies that:

$|\phi_1(t) - \phi_2(t)| \leq C|\phi_1(t) - \phi_2(t)|$, where $C < 1$. Consequently, the solution is stable in the sense of Lyapunov, as required.

3.3 Theorem: If the kernel function $K(\tau, \phi(t))$ is continuously differentiable with respect to the EEG signals $\phi(t)$ during an epileptic seizure and satisfies the condition: $\frac{\partial K(\tau,\phi(t))}{\partial \phi(t)} \leq -c < 0$, then the solution $\phi(t)$ of the integral equation 2.1 is asymptotically stable during the epileptic seizure.

Proof.

Consider a small perturbation $\delta\phi(t) = \phi(t) - \phi_0(t)$, where $\phi_0(t)$ is the steady-state solution.

Linearize the integral equation around $\phi_0(t)$, we obtain

$$\delta\phi(t) \approx \int_t^{\infty} \frac{\partial K(\tau, \phi_0(t))}{\partial \phi(t)} \delta\phi(t)\, \upsilon(t - \tau)\mu(\tau) d\tau$$

Since $\frac{\partial K(\tau, \phi(t))}{\partial \phi(t)} \leq -c < 0$, then we have $\delta\phi(t)$

$$\leq -c \int_t^{\infty} |\delta\phi(t)|\, |\upsilon(t - \tau)||\mu(\tau)| d\tau$$

This implies that $\delta\phi(t)$ decays exponentially over time, leading to asymptotic stability, as required.

3.4 Theorem: If the kernel function $K(\tau, \phi(t))$, the pre-seizure signal $\upsilon(t)$, and the seizure dynamics $\mu(\tau)$ are continuous and bounded, then the solution $\phi(t)$ is bounded for all t during the epileptic seizure.

Proof.

Since $K(\tau, \phi(t)), \upsilon(t)$, and $\mu(\tau)$ are continuous and bounded, we have

$|K(\tau, \phi(t))| \leq M_K$, $|\upsilon(t)| \leq M_\upsilon$, and $|\mu(\tau)| \leq M_\mu$ for all t and τ.

For all t, we have
$$|\phi(t)| = \left| \int_t^\infty K(\tau, \phi(t))\, \upsilon(t-\tau)\mu(\tau) d\tau \right|$$

$$\leq M_K M_\upsilon M_\mu \int_t^\infty d\tau \text{ (by assumption)}$$

Since the integral $\int_t^\infty d\tau$ converges over a finite interval, it follows that $|\phi(t)|$ is bounded for all t during the epileptic seizure, as required.

3.5 Theorem: Suppose that $K(\tau, \phi(t))$ is continuous and satisfied a Lipschitz condition during an epileptic seizure. Then the solution $\phi(t)$ of integral equation 2.1 is continuous with respect to small changes in the initial conditions during the epileptic seizure.

Proof.

Assume that $\phi_1(t)$ and $\phi_2(t)$ are solutions of 2.1 equation with slightly different initial conditions $\phi_1(t_0)$ and $\phi_2(t_0)$ during the seizure. We will show that the difference between the solutions, $|\phi_1(t) - \phi_2(t)|$, remains small for all t, implying continuity with respect to the initial conditions. Now we have:

$$|\phi_1(t) - \phi_2(t)| = \left| \int_t^\infty [K(\tau, \phi_1(t)) - K(\tau, \phi_2(t))]\, \upsilon(t-\tau)\mu(\tau) d\tau \right|$$

Since $K(\tau, \phi(t))$ satisfies a Lipschitz condition with respect to the second variable, i.e., there exists a constant $L_K > 0$ such that:

$$|\phi_1(t) - \phi_2(t)| \leq L_K \int_t^{\infty} |\phi_1(t) - \phi_2(t)|\, |\upsilon(t-\tau)||\mu(\tau)| d\tau$$

$$\leq L_K M_\upsilon M_\mu \int_t^{\infty} |\phi_1(t) - \phi_2(t)|\, d\tau \text{ by 2.3 theorem}$$

Now, assume that the difference $|\phi_1(t_0) - \phi_2(t_0)|$ is small at the initial time t_0. Therefore, the integral will remain small if $|\phi_1(t) - \phi_2(t)|$ does not grow too quickly over time. By applying Gronwall's inequality, we can deduce that the difference between the two solutions remains bounded and grows at most exponentially with time. However, since the initial difference is small, the solutions $\phi_1(t)$ and $\phi_2(t)$ will remain close for all t during the seizure. Thus, we conclude that small changes in the initial conditions of $\phi(t)$ lead to small changes in the solution $\phi(t)$ for all t, proving that the solution is continuous with respect to the initial conditions during the seizure, as required.

4. Discussion

The application of Lyapunov's stability theorem in this study provides a robust framework for analysing EEG signals during epileptic seizures. This theorem highlights the importance of understanding how small perturbations in initial conditions can lead to significant changes in model behaviour. Although this is a common assumption in stability analysis, it may not hold in highly chaotic systems like the brain, where minor variations can result in unpredictable outcomes. Integral equation-based models have been shown to offer significant advantages over traditional approaches in EEG interpretation. These models capture the transient dynamics of epileptic activity more effectively, allowing for a better understanding of the underlying brain mechanisms. However, further exploration of the parameter space, particularly regarding neural connectivity and seizure intensity, is necessary. This need for exploration suggests that our current understanding may be incomplete, potentially limiting the applicability of our findings to real-world scenarios.

While this study presents promising theoretical insights, the requirement for a continuously differentiable Lyapunov function to establish global asymptotic stability raises practical challenges. Finding such a function can be difficult, and its existence does not guarantee that the model will accurately reflect real-world dynamics. Additionally, while potential therapeutic applications of these findings have been discussed, concrete examples or evidence illustrating how these results can be translated into clinical

practice is lacking. This omission could limit the practical impact of the research. It is also important to consider that the theoretical nature of this study may lack empirical validation through clinical data or real-world EEG recordings. Such validation is crucial for establishing the reliability of the proposed models and their predictions. Furthermore, the findings may be specific to the types of seizures or conditions studied, which could restrict their generalizability to other forms of epilepsy or neurological disorders.

5. Conclusion

In this study, we investigated the mathematical stability of EEG signal models during epileptic seizures, employing integral equations to capture the complex dynamics of brain activity. Our findings indicate that the stability of these models is significantly influenced by parameters such as seizure duration, intensity, and neural connectivity. The application of Lyapunov's stability theorem has proven essential in understanding how small perturbations in initial conditions can affect model behavior. While 2.1 integral equation-based models present advantages over traditional approaches by offering a more accurate representation of transient dynamics, there are notable limitations that warrant further exploration. Specifically, the need for more complex geometries of neural networks highlights that current models may oversimplify the actual neural dynamics, potentially leading to unreliable predictions. Additionally, the exploration of the parameter space, particularly concerning neural connectivity and seizure intensity, is necessary to enhance the applicability of our findings to real-world scenarios. Moreover, the reliance on the assumption that small changes in initial conditions lead to small changes in the solution may not hold true in chaotic systems such as the brain. The challenges associated with finding continuously differentiable Lyapunov functions can impede the establishment of global asymptotic stability, and their existence does not guarantee that the model will accurately reflect real-world dynamics. While the study discusses potential therapeutic applications, concrete Examples of how these findings can be translated into clinical practice are needed to enhance their practical impact. The theoretical nature of this research also points to a lack of empirical validation through clinical data or real-world EEG recordings, which is crucial for establishing the reliability of the proposed models. Finally, it is important to note that the findings of this study may be specific to the types of seizures examined, which could limit their generalizability to other forms of epilepsy or neurological disorders.

6. Future Directions

- Clinical Validation: The future of research needs to be directed towards the validation of 2.1 integral equation models in clinical settings; most importantly, real-time EEG data is to be analysed from patients operating with epilepsy.
- Integration of Machine Learning: Integration of machine learning methods with such mathematical modelling will improve seizure prediction algorithms and give better forecasting of seizure events.
- Parameter Space Exploration and Therapeutic Applications: Clearly, the parameter space, with respect to neural connectivity and seizure intensity, has to be explored further to provide more insights into the mechanisms for stability. Extension of models to more complex geometries of neural networks or linking them to other physiological data, such as neuroimaging, may provide higher accuracy. Another fruitful direction of work is an investigation of how these models can be used to inform therapeutic interventions, including neurostimulation and personalized medication.

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